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Influence of Calorimeter Heat Transfer Gages on Aerodynamic Heating

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Nomenclature

x = space variable in the mainstream direction
u = fluid speed in the x direction
l = a typical length for the flow
Re = flow Reynolds number based on this length l and on freestream conditions
$\tau_0(x)$ = wall skin friction
ρ = local fluid mass density
$q_0(x)$ = rate of conductive heat transfer to the wall
h = local fluid total enthalpy
g = $(1 - h/h_e)$, local nondimensional total enthalpy
α = coefficient in Eq. (2) for skin friction

Subscripts

e = evaluated external to the boundary layer (in mainstream)
0 = evaluated at the wall
d = evaluated at a wall-temperature discontinuity
1 = evaluated at the front edge of a gage
2 = evaluated at the rear edge of a gage
x = based on the length x instead of on l

Analysis

The center of a calorimeter heat transfer gage such as is described by Rose and Stark[1] reaches, by design, after a short time a temperature lower than that of the surrounding model surface. It is desirable to find the effect of this near-discontinuity in surface temperature on the aerodynamic heating rate measured by the gage.

An analysis first proposed by Lighthill[2] which linearized the boundary layer energy equation is useful here. The linearization is better for small streamwise pressure gradient and for large Prandtl number. Lighthill[2] analyzed only the case of zero streamwise pressure gradient, although Illingworth[3] later analyzed the case of nonzero pressure gradient. Only the constant pressure case is used here to illustrate the influence of a nearly discontinuous wall temperature.

Solution of the linearized energy equation for a given wall-temperature distribution results[2] in the following expression for the heat transfer $q_0(x)$ to the wall:

$$\frac{q_0(x)}{\rho_e(x)u_e(x)h_e(x)} = -\frac{(\frac{2}{3})^{2/3}}{2(\frac{1}{3})!}Re^{-1/3}\left\{\frac{2\tau_0(x)}{\rho_e(x)\,u_e^2(x)}\right\}^{1/2} \times \int_{x_1=0}^{x} dg_0(x_1)\left[\int_{x_1}^{x}\frac{1}{l}\left\{\frac{2\tau_0(x_2)}{\rho_e(x_2)u_e^2(x_2)}\right\}^{1/2}dx_2\right]^{-1/3} \quad (1)$$

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(Note that $(\frac{1}{3})! = 0.8930$.) Lighthill[2] indicates that the accuracy of Eq. (1) may be improved by a suitable alteration of the constant multiplier of its right-hand side.

Application to the Gage Problem

To find the effect of the nearly discontinuous surface-temperature distribution associated with a calorimeter gage, the practical temperature distribution is approximated by one with a discontinuous decrease at the front of the gage and with a discontinuous increase at its rear edge. Such a distribution comprises a continuous one and one that is zero everywhere except on the gage where it has a constant negative value. Since the energy equation had been linearized, the results of applying Eq. (1) to each of these constituent distributions in turn may be added to give the required solution. It also is apparent from Eq. (1) that the contributions from any additional square-wave temperature distributions may be added separately. Such contributions have no upstream influence.

One should realize from the energy equation[2] that the assumed temperature discontinuity at the wall would result in an infinite wall heat transfer rate at the point of discontinuity. This invalidates Lighthill's[2] linearizing assumption at that point. However, the author considers that the solution offered here remains a good approximation for the local heating rate, except at the discontinuity where the nature of the temperature change needs exact specification and an excellent approximation for the averaged gage heating rate.

The wall skin friction for the constant pressure is assumed to be given by

$$2\tau_0(x)/\rho_e(x)u_e^2(x) = \alpha\, Re_x^{-1/2} \quad (2)$$

in which $\alpha \doteq \frac{2}{3}$, as given by Horwarth.[4] At $x = x_d$, a jump in $_0(gx)$ of g_{0d} now is allowed. Then in addition to the wall heating rate arising from the continuous part of the wall-temperature distribution, there is behind the point of discontinuity a contribution of magnitude $\delta q_0(x)$ given by

$$\frac{\delta q_0(x)}{\rho_e(x)u_e(x)h_e(x)} = -\frac{(\frac{3}{8})^{2/3}\,\alpha^{1/3}}{2(\frac{1}{3})!}Re_x^{-1/3}\left\{\frac{2\tau_0(x)}{\rho_e(x)u_e^2(x)}\right\}^{1/2} \times g_{0d}\left\{1-\left(\frac{x_d}{x}\right)^{3/4}\right\}^{-1/3} \quad (3)$$

The accuracy of Eq. (3) also may be improved by suitable alteration of the constant multiplier of its right-hand side. The infinite heating rate at the assumed temperature discontinuity is apparent from Eq. (3).

Consider now a calorimeter gage mounted between $x = x_1$ and $x = x_2$. The temperature function $g_0(x)$ is taken as one that is constant at $g_0(0^+)$ except on the gage where it is g_{01} lower than elsewhere. In front of the gage the heating rate is unchanged at $q_0(x)$, the value due to $g_0(0^+)$ alone. On the gage the fractional increase in the wall heating rate is

$$\frac{\delta q_0(x)}{q_0(x)} = \frac{g_{01}}{g_0(0^+)}\left\{1-\left(\frac{x_1}{x}\right)^{3/4}\right\}^{-1/3} \quad (4)$$

and behind the gage it is

$$\frac{\delta q_0(x)}{q_0(x)} = \frac{g_{01}}{g_0(0^+)}\left[\left\{1-\left(\frac{x_1}{x}\right)^{3/4}\right\}^{-1/3} - \left\{1-\left(\frac{x_2}{x}\right)^{3/4}\right\}^{-1/3}\right] \quad (4a)$$

In practice, an averaged heating rate of the gage is measured. Eq. (4) is seen to produce a finite average, making the present result physically acceptable. The averaged error obtainable from Eq. (4) is seen to increase directly with the temperature step and to be larger for a gage of small extent in comparison with its distance from the leading edge.

The assumption of a sharp temperature change is only justifiable, however, when the gage is not very small.

This analysis confirms the importance of measuring only the initial response of a calorimeter gage whose thermal properties differ considerably from those of the mounting surface.

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Limits on the Damping of Two-Body Gravitationally Oriented Satellites

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This paper considers the conjecture that the settling time of a strictly gravity-gradient attitude control system is limited to be of the order of the settling time of a critically damped dumbbell. The conjecture is shown to be true for a certain class of gravity-gradient systems and, in particular, for the gravity-gradient systems thus far proposed. It also is shown that gravity-gradient systems outside this class may have arbitrarily fast settling times. However, it is suggested that reliable mechanization of such rapidly settling systems may be difficult.

I. Introduction

In a previous paper,[1] the author considered the small motion damping of a two-body gravitationally oriented satellite of the type proposed by Kamm.[2] It was pointed out that this was one of several passive or semipassive gravitational schemes being considered for very reliable, long-life satellites. The common feature of all the schemes is an auxiliary inertia, either a gyro or a second rigid body, which is attached to the satellite through a dissipative joint.

For the satellite considered in Ref. 1, a bound was found on the pitch "asymptotic settling time," that is, the $1/e$ settling time of the most lightly damped mode. This was found to be $5^{1/4}/2\pi(3)^{1/2} = 0.137$ orbits. In this satellite, only a simple spring-dashpot combination was assumed between the two inertias. One immediately thinks of improving the settling time by the use of more sophisticated control, say by employing feedback. On the other hand, the bound on the $1/e$ pitch asymptotic settling time of the roll-vee gyro system of Refs. 3 and 4 is $1/(2\pi) = 0.159$ orbits—of the same order as that of the system in Ref. 1. One might conjecture that a natural bound of this order of magnitude generally exists for purely gravity-gradient schemes.

For example, suppose that the auxiliary inertia is a second rigid body. The satellite is desired stable with respect to a rotating, earth-pointing reference frame. In this frame, the satellite's natural frequency can be no higher than $(3)^{1/2}\Omega$ (Ω is the orbital frequency), corresponding to the natural frequency of a dumbbell-shaped body. Likewise, the natural frequency of any auxiliary fluid or rigid body inertia system also is less than $(3)^{1/2}\,\Omega$, and so the basic system one starts with has the lumped representation shown in Fig. 1, with two inertia systems of limited frequencies. (For convenience, a lineal rather than a rotatory model is shown.) Only if the satellite grabs onto the rotating, earth-pointing reference frame by some means other than gravity gradient can these frequencies be raised. One now provides torques between the inertias to damp the system as rapidly as possible. However, these torques are applied only *between* the inertias. It would seem likely, therefore, that the limited natural frequencies of the satellite and the auxiliary inertia would set the time scale of the oscillation. One thus might conjecture that it would be difficult to attain settling times much faster than $1/[2\pi(3)^{1/2}] = 0.092$ orbits, corresponding to the most rapid (critical) damping of a single-degree-of-freedom system with the limiting natural frequency of $(3)^{1/2}\Omega$.

It is shown in this paper that the conjecture is true for a certain class of linear systems. Systems in this class have resistive velocity-dependent torques (as defined in the next section). In addition, torques proportional to the bodies' displacements are applied between the inertias. For example, all systems with a single viscous damper and displacement proportional torques between the inertias fall within this class. The class includes, in particular, all the gravity-gradient schemes considered in Refs. 1, 2, and 5. The compliant dumbbell analyzed by Paul[6] is not of this class. However, by the methods presented, the conjecture easily is shown to hold for Paul's system as well. The conjecture thus, in fact, is true for all the gravity-gradient schemes proposed in Refs. 1–6.

Hence, to obtain a two-body gravity-gradient satellite that damps down substantially faster than these proposed systems, one must search outside the class for which the conjecture holds. Indeed, as shown in this paper by an example, it is possible to attain arbitrarily fast settling times outside of this class. However, the system shown in this paper to have arbitrarily rapid settling times also is shown to be intolerably sensitive to changes in system parameters. It goes without saying that the mechanization of any practical damping system must be reliably long-life so as not to negate the basic gravity-gradient reliability. Whether this is possible for a system outside the class considered is an open question.

II. Systems for Which the Conjecture Holds

Fourth-order system

Consider the system shown in Fig. 1. This is a schematic of the pitch motion of a two-body, gravity-gradient system. The bodies A_1 and A_2 are assumed linked at their mass centers† so that the gravity-gradient spring, $k_1 = 3(B_1 - C_1)\Omega^2$, acts between the satellite of inertia A_1 and ground of the rotating reference frame. (B_1 and C_1 are principal inertias of the satellite.) Likewise, the gravity-gradient spring $k_2 = 3(B_2 - C_2)\Omega^2$ acts between the auxiliary inertia and ground. Let a torque T that is a linear function of the velocities and displacements act between the inertias A_1 and A_2:

$$T = a_1q_1 - a_2q_2 + c_1\dot{q}_1 - c_2\dot{q}_2$$

Depending on the values of a_1, a_2, c_1, and c_2, this form of the torque can represent a variety of mechanizations. For

† As indicated in a footnote of Ref. 1, if the mass centers are noncoincident as in Kamm's vertistat, only a trivial change in the differential equations occurs. In the present case, this does not affect the results obtained.

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